# Determination of the fragmentation functions from an NLO QCD analysis of the HERMES data on pion multiplicities

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### Abstract

An NLO QCD analysis of the final HERMES data on pion multiplicities is presented and a new set of pion fragmentation functions is extracted from the best fit to the data. We have studied the so-called $[x, z]$ and $[Q^2, z]$ presentations of their data, as given by HERMES, which, in principle, should simply be two different ways of presenting the experimental data. We have based our extraction on an excellent fit to the $[Q^2, z]$ presentation of the data. We also draw attention to what appears to be a problem with the $[x, z]$ presentation of the HERMES data.

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# 1 Introduction

In the absence of charged current neutrino data, the experiments on polarized inclusive deep inelastic lepton-nucleon scattering (DIS) yield information only on the sum of quark and antiquark parton densities (PDFs), $\Delta q + \Delta \bar{q}$, and the polarized gluon density $\Delta G$. In order to extract separately $\Delta q$ and $\Delta \bar{q}$ other reactions are needed. One possibility is to use the *polarized* semi-inclusive lepton-nucleon processes (SIDIS) $l + N \rightarrow l' + h + X$, where $h$ is a detected hadron (pion, kaon, etc) in the final state. In these processes new physical quantities appear - the collinear fragmentation functions $D_{q,\bar{q}}^h(z, Q^2)$ which describe the fragmentation of quarks and antiquarks into hadrons. Due to the different fragmentation of quarks and antiquarks, the polarized parton densities $\Delta q$ and $\Delta \bar{q}$ can be determined separately from a combined QCD analysis of the data on inclusive and semi-inclusive asymmetries. The key role of the fragmentation functions for the correct determination of sea quark parton densities $\Delta \bar{q}$, especially of the polarized strange quark densitiy, was discussed in [1]. Note that the W data from RHIC give no information about the polarized strange quark density and cannot help to solve the so called "strange quark polarization puzzle" (see the second reference in [1]).

There are different sources to extract the fragmentation functions (FFs) themselves: semi-inclusive $e^+ e^-$ annihilation data, single-inclusive production of a hadron $h$ at a high transverse momentum $p_T$ in hadron-hadron collisions, unpolarized semi-inclusive DIS processes. It is important to mention that the data on hadron multiplicities in unpolarized SIDIS processes are crucial for a reliable determination of FFs, because only then can one separate $D_q^h(z, Q^2)$ from $D_{\bar{q}}^h(z, Q^2)$ (from the other processes only the sum of them can be determined). The first global analysis based on all these reactions was carried out by de Florian, Sassot, Stratmann (DSS) group [2]. As a result, the properties of the extracted set of FFs significantly differed, especially in the kaon sector, from those of the other then published sets of FFs [3] determined from analyses in which the SIDIS data have been not included. Unfortunately, the DSS FFs were based on the *unpublished* HERMES'05 SIDIS data on hadron multiplicities [4], which were not confirmed in the final HERMES data [5]. Indeed the final HERMES data differ significantly from those used in the analysis of [2] so that the FFs extracted in [2] are incorrect (see Figures 9 and 10 in [5] for LO DSS FFs, and fig. 5 in [9] for NLO DSS FFs). Moreover, in the extraction of the next-to-leading order (NLO) DSS set of FFs there was a mistake (see the correction in the Appendix in [6]) in the expression for the longitudinal gluon Wilson coefficient function in the theoretical formulae for the multiplicities (we, independently, became aware of this error recently). It has turned out that not only the DSS FFs, but all the other sets of pion and kaon FFs presented in [3] are NOT in agreement with the final HERMES [5] and the preliminary COMPASS data [7] on hadron multiplicities. In our paper [8] a theoretical analysis of these data was performed and new sets of pion fragmentation functions were extracted from the best NLO QCD fits to the data, and it was shown that they disagree significantly with the pion FFs determined from all previous analyses. Very recently de Florian at al. (DSEHS) have presented results on pion FFs obtained from their *new* global QCD analysis [9] using the final HERMES and the preliminary COMPASS data on pion multiplicities.

In our paper [8] we pointed out a possible inconsistency between the HERMES $[x, z]$ and $[Q^2, z]$ presentations of their data on pion multiplicities. Bearing in mind that the

semi-inclusive DIS hadron production processes are essential for the separation of $D_q^h$ and $D_{\bar{q}}^h$ fragmentation functions, we present in this paper a more detailed discussion of our previous analysis in which we have taken into account the mistake in the longitudinal gluon Wilson coefficient function [10] present in our previous analysis, and have used the corrected version given in [6]. Also, instead of the NLO MRST'02 set [11] we have here utilized the newer NLO MSTW'08 set [12] of unpolarized parton densities and study the influence of this on the extracted FFs.

## 2 QCD treatment of pion multiplicities

The multiplicities $M_{p(d)}^\pi(x, Q^2, z)$ of pions using a proton (deuteron) target are defined as the number of pions produced, normalized to the number of DIS events, and can be expressed in terms of the semi-inclusive cross-section $\sigma_{p(d)}^\pi$ and the inclusive cross-section $\sigma_{p(d)}^{DIS}$:

$$
\begin{aligned}
M_{p(d)}^\pi(x, Q^2, z) &= \frac{d^3 N_{p(d)}^\pi(x, Q^2, z)/dx dQ^2 dz}{d^2 N_{p(d)}^{DIS}(x, Q^2)/dx dQ^2} \Leftrightarrow \frac{d^3 \sigma_{p(d)}^\pi(x, Q^2, z)/dx dQ^2 dz}{d^2 \sigma_{p(d)}^{DIS}(x, Q^2)/dx dQ^2} \\
&= \frac{(1 + (1-y)^2) 2x F_{1p(d)}^\pi(x, Q^2, z) + 2(1-y) x F_{Lp(d)}^\pi(x, Q^2, z)}{(1 + (1-y)^2) 2x F_{1p(d)}(x, Q^2) + 2(1-y) F_{Lp(d)}(x, Q^2)}.
\end{aligned} \tag{1}
$$

In Eq. (1) $F_1^\pi, F_L^\pi$ and $F_1, F_L$ are the semi-inclusive and the usual nucleon structure functions, respectively. $F_1^\pi$ and $F_L^\pi$ are expressed in terms of the unpolarized parton densities and fragmentation functions (see [2]), while $F_1$ and $F_L$ are given purely in terms of the unpolarized parton densities.

We have assumed in our analysis that isospin $SU(2)$ symmetry for the favored and unfavored fragmentation functions holds

$$
D_u^{\pi^+}(z, Q_0^2) = D_{\bar{d}}^{\pi^+}(z, Q_0^2), \; D_{\bar{u}}^{\pi^+}(z, Q_0^2) = D_d^{\pi^+}(z, Q_0^2), \tag{2}
$$

and in addition, the following relations for the fragmentation of strange quarks into a pion:

$$
D_s^{\pi^+}(z, Q_0^2) = D_{\bar{s}}^{\pi^+}(z, Q_0^2) = D_{\bar{u}}^{\pi^+}(z, Q_0^2). \tag{3}
$$

Due to the charge conjugation invariance of the strong interactions the fragmentation functions $D_{q,\bar{q}}^{\pi^-}$ can be expressed through $D_{q,\bar{q}}^{\pi^+}$:

$$
D_{q(\bar{q})}^{\pi^-}(z, Q_0^2) = D_{\bar{q}(q)}^{\pi^+}(z, Q_0^2), \quad D_g^{\pi^-}(z, Q_0^2) = D_g^{\pi^+}(z, Q_0^2). \tag{4}
$$

As a result, we have to extract only three independent FFs ($D_u^{\pi^+}$, $D_{\bar{u}}^{\pi^+}$, $D_g^{\pi^+}$) from the NLO QCD fit to HERMES proton and deuteron data on pion multiplicities. The charm contribution to the multiplicities is not taken into account. In the theoretical analysis of the data the Mellin transform technique [10] was used to calculate the semi-inclusive $F_{1,L}^h(x, Q^2, z)$ and the usual $F_{1,L}(x, Q^2)$ nucleon structure functions in Eq.(1) from their moments. The expressions for the moments of the Wilson coefficient functions $C_{ij}^{(1)}(x, z)$ needed in these calculations can been found in [10]. As was mentioned in the Introduction, the error in the gluon Wilson coefficient, $C_{L,qg}^{(1),nm}$, was corrected. Compared to our previous

fit [8] where for the unpolarized PDFs we have used the NLO MRST'02 set [11], we use now the NLO MSTW'08 set [12], for which the strange quark density $s(x, Q^2)$ is not equal to $\bar{s}(x, Q^2)$. Note that we have chosen this set of PDFs in order to be able to compare correctly our extracted pion FFs with those of DSEHS obtained from the recent global fit [9] where the MSTW'08 set of PDFs has been used. The influence of the choice of the unpolarized densities on the extracted FFs will be discussed.

For the input FFs the following parametrization at $Q_0^2 = 1\ GeV^2$ was used:

$$zD_i^{\pi^+}(z, Q_0^2) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1 + \gamma_i(1-z)^{\delta_i}]}{B[\alpha_i + 1, \beta_i + 1] + \gamma_i B[\alpha_i + 1, \beta_i + \delta_i + 1]}, \tag{5}$$

where the parameters $\{N_i, \alpha_i, \beta_i, \gamma_i, \delta_i\}$ are free parameters to be determined from the fit to the data. Here, $i$ stands for $u, \bar{u}$ and $g$, while $B(a, b)$ denotes the Euler beta function, and the $N_i$ are chosen in such a way that they represent the contribution of $zD_i^{\pi^+}$ to the momentum sum rule.

# 3    Results of analysis

Let us discuss now our results on the pion FFs extracted from our NLO QCD fit to the HERMES proton and deuteron data on pion multiplicities, corrected for exclusive vector meson production [5]. In our study we have analyzed the $[Q^2, z]$ and $[x, z]$ presentations of the data (see Fig. 8 in [5], left column, second and third lines) for which the multiplicities do not depend on $P_{h\perp}$, were $P_{h\perp}$ is the component of the hadron momentum, $P_h$, transverse to the momentum of the virtual photon. They correspond to two-dimensional projections obtained by the HERMES group from the full HERMES data sets $[Q^2, z, P_{h\perp}]$ and $[x, z, P_{h\perp}]$, respectively. The pion multiplicities are given for 4 z-bins [0.2-0.3; 0.3-0.4; 0.4-0.6; 0.6-0.8] as functions of the mean value of $Q^2$, $< Q^2 >$, of each individual $Q^2$ bin for the $[Q^2, z]$ presentation or as functions of the mean value of $x$, $< x >$, of each individual $x$ bin for the $[x, z]$ presentation. Note that for the $[Q^2, z]$ presentation there is no binning in $x$. This means that the multiplicity measured in a given $Q^2$ bin, $Q_{min}^2 \leq Q^2 \leq Q_{max}^2$, corresponds to the summing over all possible values of $x$ belonging to the strip in the $\{x - Q^2\}$ plane, bounded by $Q_{min}^2$, $Q_{max}^2$ and the kinematics of the HERMES experiment. And vice versa, for the $[x, z]$ presentation, there is no binning in $Q^2$, and the multiplicity measured for each $x$ bin corresponds to all possible values of $Q^2$ belonging to the $\{x, Q^2\}$ strip fixed by the boundaries of the $x$ bin and the kinematics of the HERMES experiment. Thus, in principle, in the theoretical calculation of the pion multiplicities one has to integrate the semi-inclusive and inclusive cross section on RHS side of Eq. (1) over the $x$ and $Q^2$ regions corresponding to each $Q^2$ bin for the $[Q^2, z]$ presentation or to each $x$ bin for the $[x, z]$ one. It turns out however, that replacing $x$ and $Q^2$ by their mean values $< x >$ and $< Q^2 >$ in the calculation of the multiplicities leads to very small difference. Further details are presented later.

The total number of the $\pi^+$ and $\pi^-$ data points for each of the presentations is 144, 72 for $\pi^+$ and 72 for $\pi^-$ data. In the case of $[Q^2, z]$ presentation of the data a good fit to the proton and deuteron data is achieved, $\chi^2/\mathrm{d.o.f} = 123.95/132 = 0.94$ for 144 experimental points and 12 free parameters. The errors used in the fit are quadratic combinations of the statistical and point-to-point systematic errors. We have found that the description
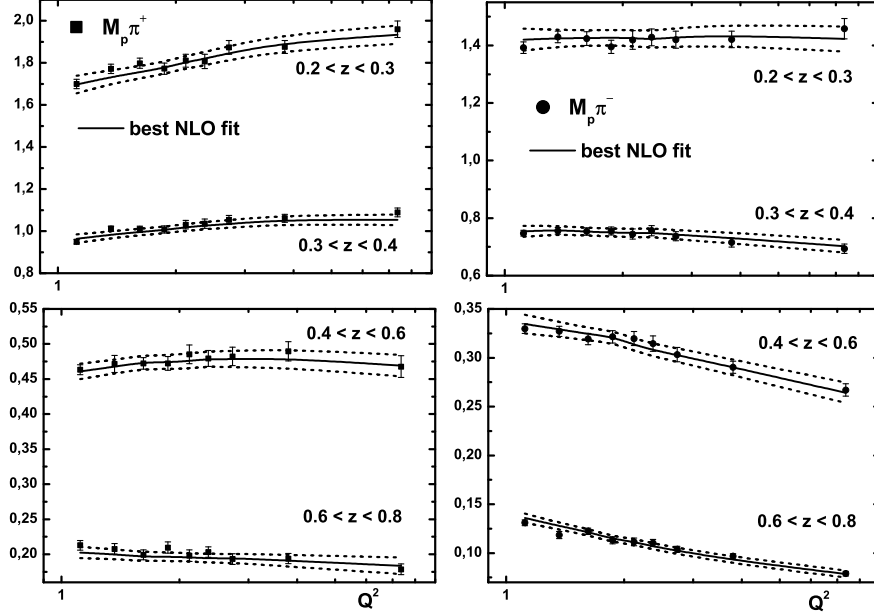
Figure 1: Comparison of HERMES $[Q^2, z]$ *proton* data on $\pi^+$ (left) and $\pi^-$ multiplicities (right) with the best NLO fit curves. The error bands (the area between the dot curves) correspond to uncertainty estimates at 68% C.L. The errors of the data are *total*, statistical and systematic taken in quadrature.

of the proton data (the mean value of $\chi^2$ per point is equal to 0.83 for $\pi^+$ and 0.65 for $\pi^-$ multiplicities) is better than that of the deuteron data (where the mean value of $\chi^2$ per point is equal to 0.98 for $\pi^+$ as well as for $\pi^-$ multiplicities). The quality of the fit to the data is illustrated in Fig. 1 (for the proton target) and Fig. 2 (for the deuteron target). The error bands (the area between the dot curves) correspond to uncertainty estimates at 68% C.L. Note that the vertical scale is linear, not logarithmic.

**TABLE I.** The parameters of the NLO input FFs at $Q^2 = 1\ GeV^2$ obtained from the best fit to the data. The parameters marked by (*) are fixed.

| Flavor | N | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|---|
| $u$ | $0.278 \pm 0.016$ | $0.276 \pm 0.192$ | $0.188 \pm 0.187$ | $7.64 \pm 1.62$ | $3.09 \pm 0.40$ |
| $\bar{u}$ | $0.153 \pm 0.016$ | $0.282 \pm 0.251$ | $1^*$ | $9.18 \pm 4.12$ | $3.85 \pm 0.45$ |
| g | $0.113 \pm 0.005$ | $12.70 \pm 5.64$ | $14.39 \pm 6.35$ | $0^*$ | $-$ |

The values for the parameters of the input FFs (5) obtained from the best fit to the data are presented in Table I. It turned out during the fit that there was a slight preference for the parameter $\beta_{\bar{u}}$ to go to the somewhat unphysical limit zero, but the value of $\chi^2$, as well as the values of $D_{\bar{u}}^{\pi^+}(z)$ for the measured range of z, $z \in [0.2,\ 0.8]$, practically do not change for fixed values of $\beta_{\bar{u}}$ in the range $[0,\ 2]$. That is why it was fixed at the reasonable value $\beta_{\bar{u}} = 1$. Also, because of the small $Q^2$ range of the HERMES data, a simpler parametrization for the gluon FF $D_g^{\pi^+}(z)$ was used with only three parameters
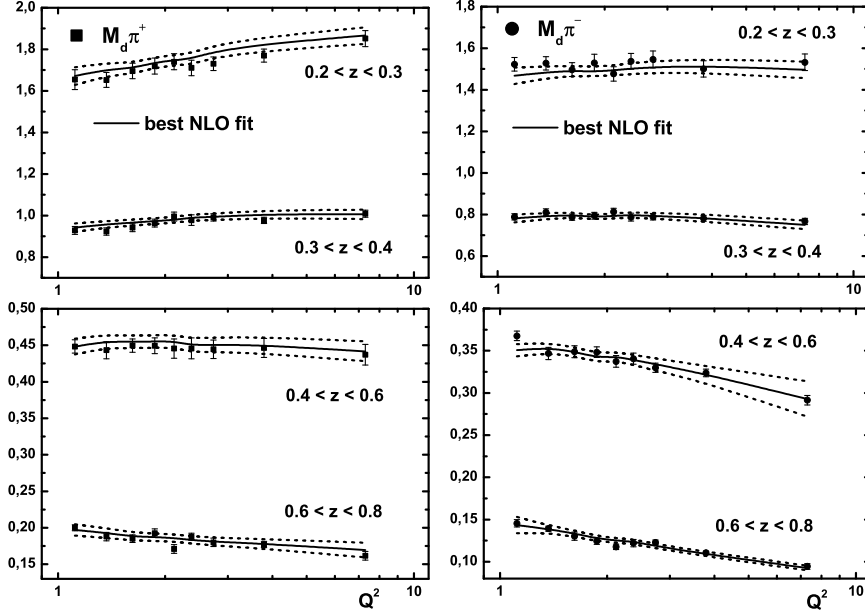
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Figure 2: Comparison of HERMES $[Q^2, z]$ *deuteron* data on $\pi^+$ (left) and $\pi^-$ multiplicities (right) with the best NLO fit curves. The error bands (the area between the dot curves) correspond to uncertainty estimates at 68% C.L. The errors of the data are *total*, statistical and systematic taken in quadrature.

and $\gamma_g = 0$.

The extracted pion FFs from the fit to HERMES $[Q^2, z]$ data on pion multiplicities are presented in Fig. 3 along with their error bands corresponding to the uncertainty estimates at 68% C.L, and compared to those determined recently by DSEHS from their global analysis [9] which also made use of the HERMES $[Q^2, z]$ data. In Fig. 3 the error band for the gluon fragmentation function corresponding to $\Delta\chi^2 = 1$ (the black shaded band) is also presented. The corresponding error bands for the other FFs are not presented because they are very narrow and practically not visible. The fragmentation functions are plotted for the mean value of $Q^2$ for the HERMES data, $Q^2 = 2.5$ GeV$^2$, and for the measured z region [0.2-0.8]. One can see from Fig. 3 that our (LSS) pion FFs $D_u^{\pi^+}(z)$ and $D_{\bar{u}}^{\pi^+}(z)$ are close to those of DSEHS (solid curves). In the DSEHS analysis the equality (3) for the fragmentation of the strange and $\bar{u}$ quarks into pion is not assumed. As a result, the extracted fragmentation functions for the strange quark, $D_{\bar{s}}^{\pi^+}(z)$, differ a little in the z range $0.2 < z < 0.35$. The main difference between the extracted FFs is for the gluons. This is not unexpected bearing in mind that an accurate determination of the qluon fragmentation function requires data covering a large range in $Q^2$ and that for the semi-inclusive DIS processes the range for the HERMES $[Q^2, z]$ data is small: $1.1 < Q^2 < 7.4$ GeV$^2$.

We have tried to get a feeling for the dependence of the results on the unpolarized PDFs used in the analysis, and find that when the MRST'02 set is used instead of the NLO MSTW'08 the description of the data is slightly worse, with a value of $\chi^2/$d.o.f equal to 1.00 (0.94 for MSTW'08 PDFs). In Fig. 4 we illustrate the sensitivity of the extracted
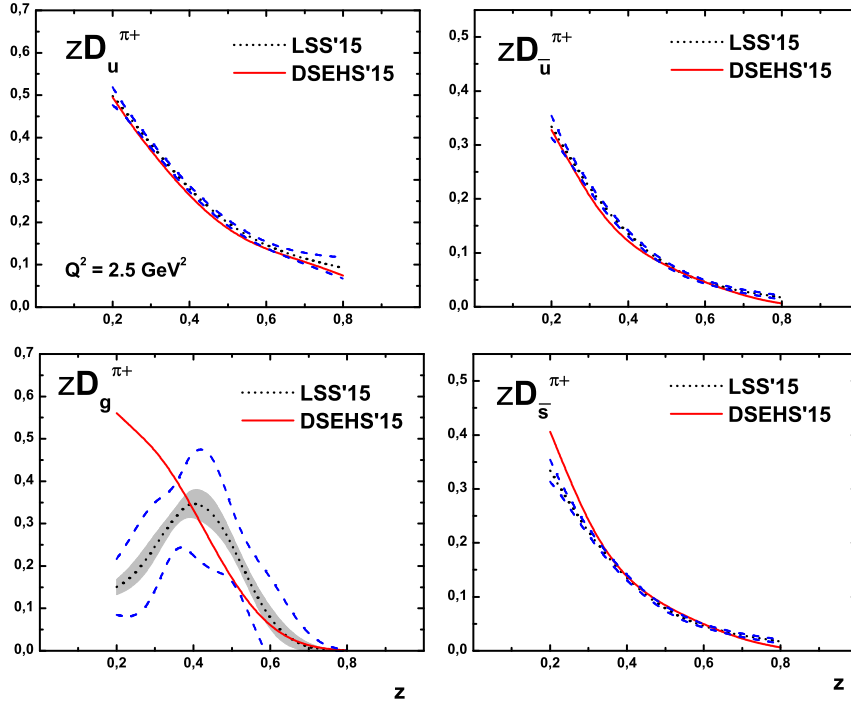
Figure 3: Comparison between our pion FFs at $Q^2 = 2.5$ GeV$^2$ along with the uncertainty estimates at 68% C.L. (the area between the dashed curves) and those of DSEHS (solid curves). For the gluon fragmentation function the uncertainty corresponding to $\Delta\chi^2 = 1$ (the black shaded band) is also presented.

pion FFs to the use of different sets of NLO unpolarized PDFs, in our case MWST'08 and MRST'02. The corresponding FFs $D_u^{\pi^+}(z)$ and $D_{\bar{u}}^{\pi^+}(z)$ are not shown because the differences between them are so small that they are not visible. Instead, for them, the error bands corresponding to the uncertainty estimates at 68% C.L. for $D_u^{\pi^+}(\text{MRST}'08)$ (the black solid curves) and $D_{\bar{u}}^{\pi^+}(z)$ (the dashed curves), respectively, are plotted in Fig. 4(left), and compared with the differences $\Delta D_u^{\pi^+}$ and $\Delta D_{\bar{u}}^{\pi^+}$, short dash and dash dot dot curves, respectively, where

$$\Delta D_{u,\bar{u}}^{\pi^+} = D_{u,\bar{u}}^{\pi^+}(\text{MRST}'02) - \text{D}_{u,\bar{u}}^{\pi^+}(\text{MSTW}'08). \tag{6}$$

Also for the gluons the difference is very small, but is at least visible, as shown in Fig. 4(right). Note that because of the large uncertainty in the determination of the gluon FF in Fig. 4(right) only the error band corresponding to $\Delta\chi^2 = 1$ is presented. As seen from Fig. 4, the central values of the fragmentation functions corresponding to the use of MRST'02 PDFs lie entirely within the error bands for FFs corresponding to the use of MSTW'08 set of PDFs. The fact that a choice of PDFs other than the MSTW'08 set does not substantially alter the results of the global fit was mentioned also in [9]. Thus, to summarize, the extraction of the FFs is weakly dependent on the choice of unpolarized PDFs.
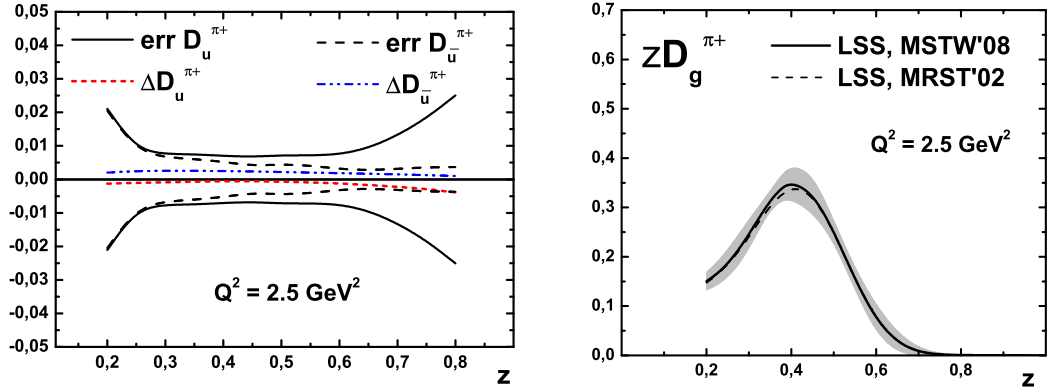
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Figure 4: Sensitivity of the extracted favored and unfavored FFs (left), and gluon FF (right) to the choice of the set of unpolarized PDFs (see the text). Note the extremely small scale of the vertical axis in Fig. 4(left).

Using the extracted FFs from the HERMES data on multiplicities in the $[Q^2, z]$ presentation we have calculated the multiplicities at the kinematic points for the data in the $[x, z]$ presentation. The obtained value for $\chi^2$ is huge, 2187.8 for 144 experimental points (recall that the corresponding value of $\chi^2$ for the $[Q^2, z]$ data is 123.95). The results are shown in Fig. 5 for the proton and in Fig. 6 for the deuteron target. The theoretical multiplicities are presented along with their uncertainty estimates corresponding to 68% C.L. As seen from the figures, the discrepancy is very large for both the proton and deuteron targets for the first two z-bins [0.2-0.3] and [0.3-0.4], as well as at lowest x, for all z-bins. In our opinion such a significant discrepancy is totally unphysical. In an attempt to understand this we have tried to fit the HERMES $[x, z]$ data *directly* and found that we cannot obtain a fit with a reasonable $\chi^2$ using different input parametrizations for the fragmentation functions. In addition, for some of the parameters we obtain values in the non-physical region. It is clear that the trend of the data in the small $x$ region is different not only from that of the QCD predictions in this region, but also from the rest of the data points in each z bin. Consequently we decided to perform a NLO QCD fit to $[x, z]$ data after removing the three lowest $x$ data points for every z bin. The total number of removed data points for $\pi^+$ and $\pi^-$ multiplicities is 48 for which the contribution to $\chi^2$ above is 1470 (30.6 per point). In the fit to the rest of the data (96 points; we will refer to this data set as the "cut" $[x, z]$ data) we have used for the input FFs the parametrization given in Eq. (5). Not unexpectedly it turned out that the input parameters for the gluon FF can not be fixed well from the fit, so for them we have used the parameters obtained from the fit to the $[Q^2, z]$ data (see Table I). The following value for $\chi^2$/d.o.f, $\chi^2$/d.o.f = 179.37/87 = 2.06 for 96 experimental points and 9 free parameters, is achieved in the fit. The results of the best fit are shown in Fig. 5 for a proton target and Fig. 6 for the deuteron one (solid curves). Their continuation to the low x region where the data points were removed from the fit, is indicated by the dashed curves. The quality of this fit is illustrated in Table II, and compared to the quality achieved in the fit to the $[Q^2, z]$ data. It follows from the $\chi^2$ values, presented in Table II, that the description of the $[Q^2, z]$ data
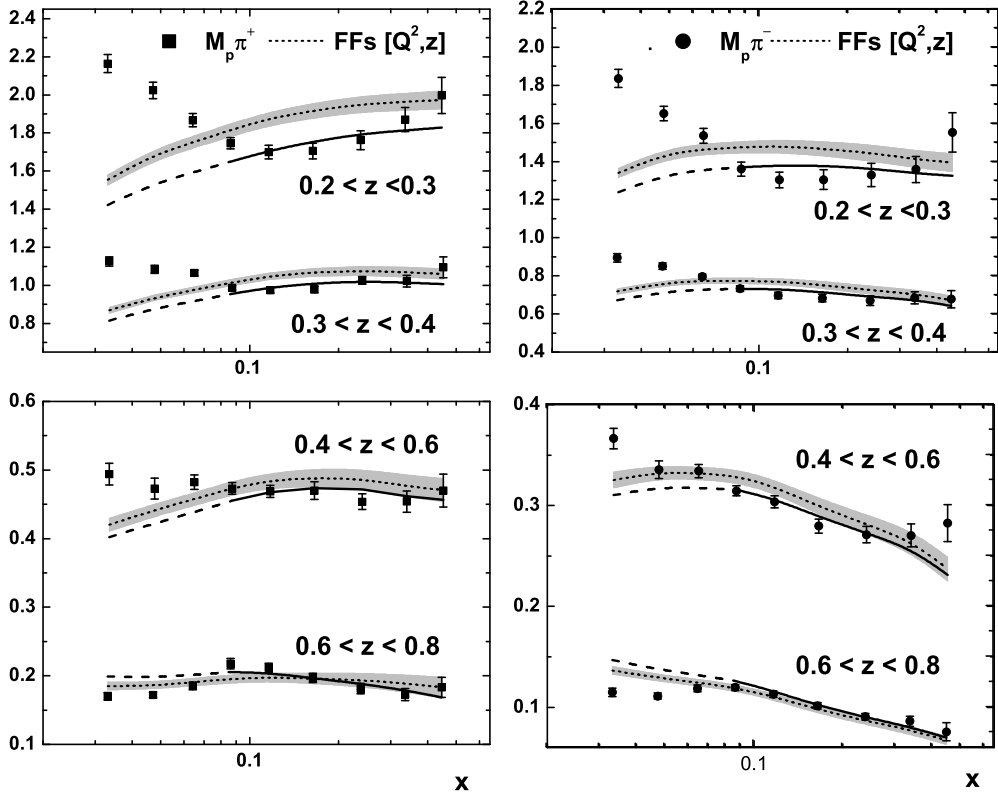
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Figure 5: Comparison of HERMES $[x, z]$ *proton* data on $\pi^+$ (left) and $\pi^-$ multiplicities (right) with the multiplicities at the same kinematic points calculated by our FFs extracted from HERMES $[Q^2, z]$ data (dot curves with the bands corresponding to uncertainty estimates at 68% C.L). The errors of the data are *total*, statistical and systematic taken in quadrature. The solid curves correspond to the best *fit* to the cut $[x, z]$ data (see the text).

is much better than that of the $[x, z]$ data even after removing a third of the data points.

The extracted pion favored and unfavored FFs from the fit to the HERMES cut $[x, z]$ data on pion multiplicities are presented in Fig. 7 and compared to those determined from the fit to $[Q^2, z]$ data, for which the error bands corresponding to the uncertainty estimates at 68% C.L. are also presented. Recall that the gluon FF is the same for both the representations of the data and it is shown in Fig. 3. As seen from Fig. 7, the central values of the favored pion FF (solid curve) extracted from the cut $[x, z]$ data are systematically smaller than those extracted from $[Q^2, z]$ data, and in the z region $[0.2, 0.4]$ the corresponding curve lies outside the error band. The central values of unfavored pion FF extracted from the cut $[x, z]$ data are also systematically smaller then those extracted from $[Q^2, z]$ data, however, the corresponding curve lies within the error band. It is important to mention, however, that from the calculation of the multiplicities at the kinematic points for the data in the $[Q^2, z]$ presentation, using the extracted FFs from the fit to the cut $[x, z]$ data, we obtain for $\chi^2$ the value 665.2 which is more than five times larger than the value 123.95 achieved in the direct fit to the $[Q^2, z]$ data.
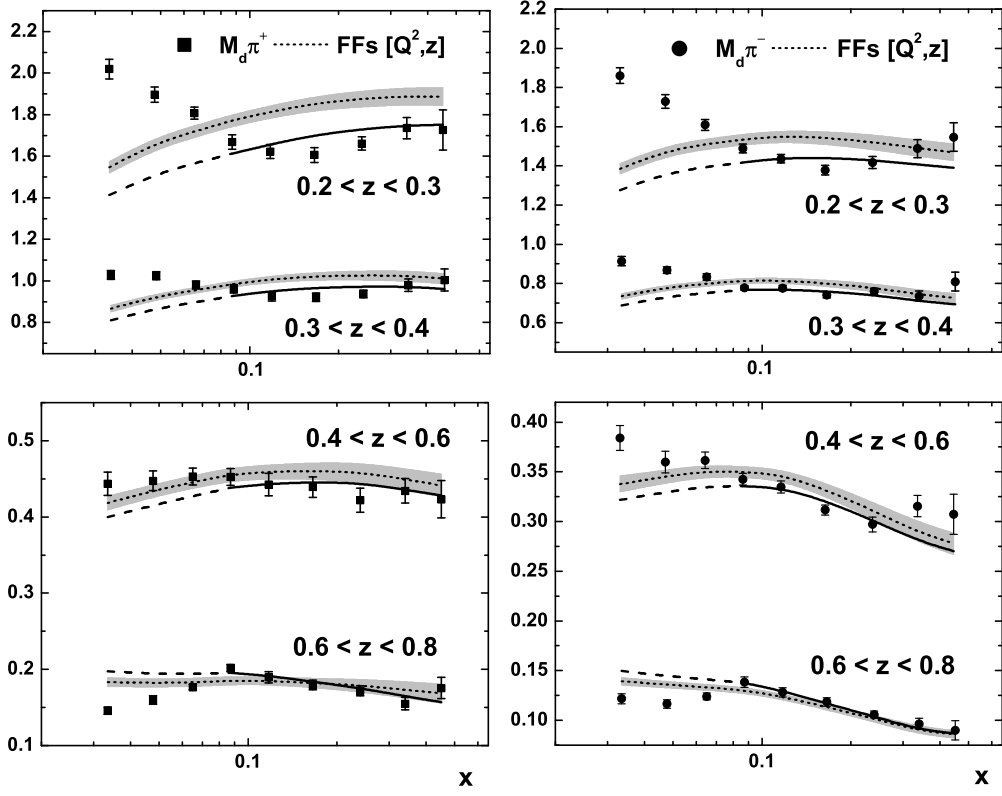
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Figure 6: As in Fig. 5 but for a deuteron target.

**TABLE II.** $\chi^2$ per point values for the pion multiplicities obtained from the fits to $[Q^2, z]$ and the cut $[x, z]$ data.

|  | $[Q^2, z]$ fit | $[x, z]$ fit |
|---|---|---|
| $M_p^{\pi^+}$ | 0.83 | 1.94 |
| $M_p^{\pi^-}$ | 0.65 | 1.58 |
| $M_d^{\pi^+}$ | 0.98 | 1.63 |
| $M_d^{\pi^-}$ | 0.98 | 2.33 |

Note that in all our NLO QCD calculations of the $[x, z]$ pion multiplicities we have used for $x$ and $Q^2$ their mean values $< x >$ and $< Q^2 >$ as given in the HERMES data tables [5]. We have checked that in NLO QCD the pion multiplicities calculated at the average kinematics $\{< x >, < Q^2 >\}$ coincide extremely closely (to better than 1%) with the average multiplicities calculated using the expression, Eq. (1), given in the recent HERMES paper [13] as applied to the NLO semi-inclusive and DIS cross sections (see the remark [14]). This fact is very important because it means that the huge time consuming the computer calculations involved in using the above mentioned expression in fitting the data on the average multiplicities can be significantly reduced if the NLO

QCD multiplicities are calculated at the corresponding mean values $< x >$ and $< Q^2 >$.

Finally we would like to underline that our NLO QCD analysis of the HERMES $[x, z]$ data supports the assertion of Stolarski [15], based on a LO QCD analysis, that the increase of magnitude of the HERMES pion multiplicity sum as $x$ decreases in the region $x < 0.1$, is difficult to reconcile with perturbative QCD. While in [15] the argument is presented for a deuteron target, our observation is that it holds for both proton and deuteron targets.
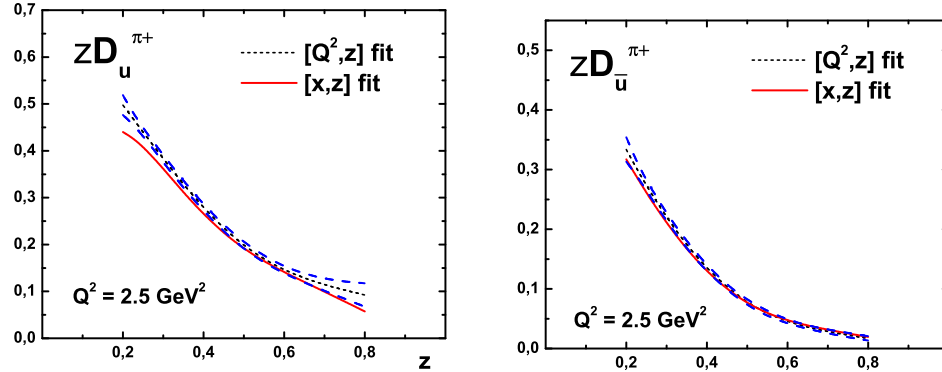


Figure 7: Comparison between favored (left) and unfavored (right) fragmentation functions extracted from $[Q^2, z]$ (dot curves) and the cut $[x, z]$ HERMES data on multiplicities (solid curves). The dashed curves mark the error bands for FFs determined from the $[Q^2, z]$ data.

# 4  Summary

The publication by HERMES of the final version of their data on pion multiplicities on protons and deuterons has profound implications for our understanding of the pion fragmentation functions.

1) The fact that the final data are significantly different from the preliminary data means that the oft utilized DSS FFs [2], which were based on the preliminary data, are incorrect.

2) We have studied the two-dimensional projections of the final HERMES data, the so-called $[x, z]$ and $[Q^2, z]$ formats, presented by the HERMES group.

a) With the pion FFs, parametrized in a standard way, and respecting isospin invariance, we have found an excellent fit to the $[Q^2, z]$ presentation of the data and extracted a new set of NLO pion FFs. Except for the gluon fragmentation function, our new pion FFs are very similar indeed to those obtained recently by the DSEHS group [9] using, in their global analysis, the $[Q^2, z]$ HERMES data.

b) On the contrary, no reasonable NLO QCD fit could be achieved to the $[x, z]$ presentation of the data. We have found that an adequate fit to the $[x, z]$ data is only possible if we cut points with $x < 0.075$ from the data which means that a third of the data

points is removed. However, even with these cuts, the quality of the description of the $[Q^2, z]$ data is much better than that achieved for the cut $[x, z]$ data. While the extracted unfavored pion FF lies within the error band corresponding to the unfavored pion FF extracted from the fit to the $[Q^2, z]$ data, the favored pion FF$[x, z]$ is systematically smaller than favored FF$[Q^2, z]$ and is outside of its error band in the region $0.2 < z < 0.4$.

3) We have found that the trend of the data in the HERMES $[x, z]$ presentation of their data, where the magnitude of the pion multiplicitities in the region $x < 0.1$ increases as $x$ decreases, is totally at variance with the trend of the NLO QCD predictions. This suggests that possibly there is a problem with the HERMES $[x, z]$ presentation of their data, and emphasizes the need for new data on the hadron multiplicities. We thus await with great interest the publication of the final COMPASS data on the pion multiplicities.

## Acknowledgments

# References

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[14] Note that Eq. (1) for the multiplicities in the $[x, z]$ presentation of the HERMES data as given in [13], is not correct. It disagrees with the verbal prescription given in [13] as to how to use the HERMES database on the multiplicities, in order to compare the experimantal value of the $[x, z]$ pion multiplicity measured in a given $x$-bin to the theory predictions for the same bin. According to this prescription one has to integrate the numerator and denominator in Eq. (1) over all $x$ values lying between the lower and upper boundaries of the bin.

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